

7.2: Adding and Subtracting Rational Expressions, Simplifying Complex Fractions

Adding and subtracting rational expressions is quite similar to adding and subtracting rational numbers (fractions).

Adding and subtracting rational expressions with a common denominator

Let's first review how to add or subtract fractions when the denominator is the same.

Recall that if the denominator is the same, it stays the same, and then we add or subtract the numerators. Finally, we simplify the result.

Example a.

The diagram illustrates the process of adding two fractions with a common denominator. At the top, an orange cloud contains the text "Add or subtract numerators." with arrows pointing to the numerators 7 and 1 in the fraction $\frac{7}{12} + \frac{1}{12}$. Below this, the equation is shown as $\frac{7}{12} + \frac{1}{12} = \frac{6}{12} = \frac{(3)(2)}{(3)(2)(2)} = \frac{1}{2}$. A green cloud at the bottom contains the text "Denominator stays the same." with arrows pointing to the denominator 12 in both fractions and the denominator 12 in the intermediate result. A red box on the right contains the text "Finish by factoring and cancelling common factors." with an arrow pointing to the cancellation of the 3 and 2 factors in the denominator of the final fraction.

$$\frac{7}{12} + \frac{1}{12} = \frac{6}{12} = \frac{(3)(2)}{(3)(2)(2)} = \frac{1}{2}$$

Adding and subtracting rational expressions works in the same manner. If the denominator is the same, it stays the same, then we add or subtract numerators. Finally, we simplify the resulting expression completely.

Example b.

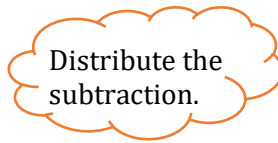
The diagram illustrates the process of adding two rational expressions with a common denominator. At the top, an orange cloud contains the text "Add or subtract numerators." with arrows pointing to the numerators x and y in the expression $\frac{x}{x^2 - y^2} + \frac{y}{x^2 - y^2}$. Below this, the equation is shown as $\frac{x}{x^2 - y^2} + \frac{y}{x^2 - y^2} = \frac{x + y}{x^2 - y^2} = \frac{(x + y)}{(x - y)(x + y)} = \frac{1}{x - y}$. A green cloud at the bottom contains the text "Denominator stays the same." with arrows pointing to the denominator $x^2 - y^2$ in both fractions and the denominator $x^2 - y^2$ in the intermediate result. A red box on the right contains the text "Finish by factoring and cancelling common factors." with an arrow pointing to the cancellation of the $(x + y)$ factors in the denominator of the final fraction.

$$\frac{x}{x^2 - y^2} + \frac{y}{x^2 - y^2} = \frac{x + y}{x^2 - y^2} = \frac{(x + y)}{(x - y)(x + y)} = \frac{1}{x - y}$$

Notice that when we simplify rational expressions, we have to first check to see if the polynomial is factorable.

Let's try one more example.

Example c.



$$\frac{2x - 4}{x + 2} - \frac{x - 6}{x + 2} = \frac{2x - 4 - (x - 6)}{x + 2} = \frac{2x - 4 - x + 6}{x + 2} = \frac{x + 2}{x + 2} = 1$$

Adding and subtracting rational expressions that do not have a common denominator

What happens when the denominator is different? In that case, we need to find a common denominator before we add or subtract, and rewrite our fractions to all have the same denominator.

Example a. To add

$$\frac{3}{4} + \frac{1}{3}$$

We must first rewrite each fraction so that it has the common denominator of 12. The first fraction is multiplied by $\frac{3}{3}$. The second fraction is multiplied by $\frac{4}{4}$. Then we can add:

$$\frac{3}{4} + \frac{1}{3} = \frac{3(3)}{4(3)} + \frac{1(4)}{3(4)} = \frac{9}{12} + \frac{4}{12} = \frac{13}{12}$$

Notice that in this simple example, the common denominator is gotten by multiplying the two denominators together: $4 \times 3 = 12$. Then we can just multiply each fraction top and bottom by the other denominator.

Example b. The rational expressions below work in the same way:

$$\frac{5}{x + 3} + \frac{1}{x + 2}$$

The common denominator is $(x + 3)(x + 2)$, gotten by multiplying the two denominators, and we can rewrite each fraction by multiplying by the other denominator:

$$\begin{aligned} & \frac{5}{x + 3} + \frac{1}{x + 2} \\ & \left[\begin{array}{l} \text{The first} \\ \text{fraction is} \\ \text{multiplied by} \\ \frac{x+2}{x+2} \end{array} \right] = \frac{5(x+2)}{(x+3)(x+2)} + \frac{1(x+3)}{(x+2)(x+3)} \left[\begin{array}{l} \text{The second} \\ \text{fraction is} \\ \text{multiplied by} \\ \frac{x+3}{x+3} \end{array} \right] \\ & = \frac{5(x+2) + 1(x+3)}{(x+2)(x+3)} \\ & = \frac{5x + 10 + x + 3}{(x+2)(x+3)} = \frac{6x + 13}{(x+2)(x+3)} \end{aligned}$$

Why can't I just **add 2** to the first fraction and **add 3** to the second one?

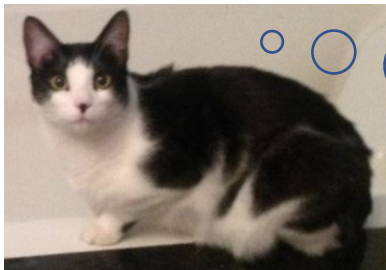
$$\frac{5}{x+3} + \frac{1}{x+2} = \frac{5+2}{x+3+2} + \frac{1+3}{x+2+3} =$$

$$\frac{7}{x+5} + \frac{4}{x+5} = \frac{11}{x+5}$$


It's only okay to **multiply** a fraction top and bottom by the same number because we are multiplying by a form of 1, so we don't change the fraction. For example, if we have the fraction $\frac{1}{2}$ and we multiply the numerator and denominator by 2, we get $\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$, which is still equivalent to $\frac{1}{2}$. This is because multiplying by $\frac{2}{2}$ is the same as multiplying by 1.

Similarly, multiplying by $\frac{x+2}{x+2}$ is the same as multiplying by 1.

But **adding** to the numerator and denominator of a fraction results in a different fraction. For example, if we have the fraction $\frac{1}{2}$ and add 2 to the numerator and denominator, we get $\frac{1+2}{2+2} = \frac{3}{4}$, which is a totally different fraction!



Oh! I get it!

$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4} \text{ is true}$$

$$\text{But } \frac{1}{2} = \frac{1+2}{2+2} = \frac{3}{4} \text{ is not true!}$$

The Least Common Denominator (LCD)

You can always find a common denominator by multiplying all the denominators. But this method will not always result in the lowest, or least, common denominator.

Example c. To add and subtract the following:

$$\frac{3}{14} - \frac{1}{9} + \frac{2}{21}$$

We first need a common denominator. If we multiply $14 \times 9 \times 21 = 2,646$, we get a very large common denominator that may be hard to work with. It may be much easier if we can find a smaller common denominator.

To find the least (smallest) common denominator (LCD),

1. Factor each denominator.
2. Find the *most* number of times each factor occurs in each denominator.
3. Multiply together each unique factor you found in each denominator, using each factor the most amount of times it occurs in a particular denominator.

In our example, above, we have the denominators 14, 9 and 21.

Step 1: factor each denominator

$$14 = 2 \times 7$$

$$9 = 3 \times 3$$

$$21 = 3 \times 7$$

So, we have 3 unique factors, 7, 2, and 3.

Step 2: 7 occurs as a factor once in 14, no times in 9 and once in 21. So, the most amount of times 7 occurs as a factor is *once*.

2 occurs as a factor once in 14, no times in 9 and no times in 21. So, the most amount of times that 2 occurs as a factor is *once*.

3 occurs no times as a factor in 14, twice as a factor in 9, and once as a factor in 21. So, the most amount of times that 3 occurs as a factor is *twice*.

Step 3: Therefore, the LCD is the product: $7 \times 2 \times 3 \times 3 = 126$

When finding the LCD for rational expressions, it works the same way:

We first factor the denominators completely, then look at the most amount of times each individual factor occurs per any one denominator. The product of these factors is the LCD.

Example d.

$$\frac{x + 4}{2x + 10} + \frac{5}{x^2 + 10x + 25}$$

To find the LCD, we must first factor each denominator.

$$2x + 10 = 2(x + 5)$$

$$x^2 + 10x + 25 = (x + 5)(x + 5)$$

We have 2 unique factors here, 2 and $(x + 5)$. We have to find the most amount of times each factor occurs per any one denominator.

2 occurs once in $2(x + 5)$, and no times in $(x + 5)(x + 5)$. So the most amount of times that 2 occurs is once.

$(x + 5)$ occurs once in $2(x + 5)$ and twice as a factor in $(x + 5)(x + 5)$. So, the most amount of times that $(x + 5)$ occurs as a factor is twice.

We multiply these factors together to get the LCD: $2(x + 5)(x + 5)$, which we can also write as $2(x + 5)^2$

Now that we have discussed how to find the LCD, let's finish discussing how to add or subtract rational expressions whose denominators are different.

e. Perform the indicated operation:

$$\frac{3}{14} - \frac{1}{9} + \frac{2}{21}$$

From example c, we already know that the LCD of this expression is 126. Once we find the LCD, we rewrite each fraction with the LCD, by using equivalent fractions

$$\frac{3}{14} - \frac{1}{9} + \frac{2}{21} = \frac{\quad}{126} - \frac{\quad}{126} + \frac{\quad}{126}$$

We do this by asking ourselves the following question: What factor did I introduce to my old denominator to get my new one? Whatever that factor is, we have to multiply our numerator by it to find equivalent fractions.

Let's start with $\frac{3}{14} = \frac{\quad}{126}$. To get from 14 to 126, we have to multiply by 9. Multiply the numerator of our old fraction by the same number, and that gives us:

$$\frac{3 \times 9}{14 \times 9} = \frac{27}{126}$$

Same for $\frac{1}{9} = \frac{\quad}{126}$. To get from 9 to 126, we have to multiply by 14. Multiply the numerator of our old fraction by 14 as well, and that gives us

$$\frac{1 \times 14}{9 \times 14} = \frac{14}{126}$$

Last fraction, $\frac{2}{21} = \frac{\quad}{126}$. To get from 21 to 126, we have to multiply by 6. Multiply the numerator of our old fraction by 6 as well, and that gives us

$$\frac{2 \times 6}{21 \times 6} = \frac{12}{126}$$

So that,
$$\frac{3}{14} - \frac{1}{9} + \frac{2}{21} = \frac{27}{126} - \frac{14}{126} + \frac{12}{126}$$

Once the denominators are the same, they stay the same, and we add or subtract numerators as the problem tells us:
$$\frac{3}{14} - \frac{1}{9} + \frac{2}{21} = \frac{27}{126} - \frac{14}{126} + \frac{12}{126} = \frac{25}{126}$$

And then we simplify completely. 25 and 126 don't share any factors that are the same except for 1, so the answer is already simplified completely, and we are done. Let's look at some examples of adding or subtracting rational expressions.

In general, for adding or subtracting rational expressions, it is a 4 step process:

STEP 1: Find the Least Common Denominator (LCD)

STEP 2: Rewrite the numerator using the LCD

STEP 3: Add or subtract numerator, keeping the denominator the same.

STEP 4: Factor numerator again (if possible) as to simplify completely

Ex: Perform the indicated operation(s), simplify completely:

d. $\frac{1}{5a} - \frac{3}{a^2}$

STEP 1: LCD

$$5a = 5 \times a$$

$$a^2 = a \times a$$

$$\text{LCD} = 5a^2$$

$$\frac{1}{5a} - \frac{3}{a^2} = \frac{\quad}{5a^2} - \frac{\quad}{5a^2}$$

STEP 2: rewrite the numerator

To get from $5a$ to $5a^2$, we have to introduce a factor of a .

To get from a^2 to $5a^2$ we have to introduce a factor of 5

$$\frac{1 \times a}{5a \times a} - \frac{3 \times 5}{a^2 \times 5} = \frac{a}{5a^2} - \frac{15}{5a^2}$$

$$\frac{a}{5a^2} - \frac{15}{5a^2} = \frac{a-15}{5a^2}$$

STEP 3: Add or subtract numerator

$$\frac{a-15}{5a^2}$$

STEP 4: Simplify. This cannot be simplified.

$$e. \frac{4}{2x+12} - \frac{6}{x^2-36}$$

$$\frac{4}{2(x+6)} - \frac{6}{(x-6)(x+6)} = \frac{\quad}{2(x-6)(x+6)} - \frac{\quad}{2(x-6)(x+6)}$$

$$\frac{4 \times (x-6)}{2(x+6) \times (x-6)} - \frac{6 \times 2}{(x-6)(x+6) \times 2} = \frac{4x-24}{2(x-6)(x+6)} - \frac{12}{2(x-6)(x+6)}$$

$$\frac{4x-24-12}{2(x-6)(x+6)} = \frac{4x-36}{2(x-6)(x+6)}$$

$$\frac{4(x-9)}{2(x-6)(x+6)} = \frac{2(x-9)}{(x-6)(x+6)}$$

$$f. \frac{6x+5}{5x-25} - \frac{x+2}{x-5}$$

$$\frac{6x+5}{5(x-5)} - \frac{x+2}{x-5} = \frac{\quad}{5(x-5)} - \frac{\quad}{5(x-5)}$$

STEP 1: LCD

$$2x + 12 = 2(x + 6)$$

$$x^2 - 36 = (x - 6)(x + 6)$$

$$\text{LCD} = 2(x - 6)(x + 6)$$

STEP 2: rewrite the numerator

To get from $2(x+6)$ to $2(x-6)(x+6)$, we have to introduce a factor of $(x - 6)$

To get from $(x - 6)(x + 6)$ to $2(x - 6)(x + 6)$, we have to introduce a factor of 2.

STEP 3: Add or subtract the numerator

STEP 4: Simplify completely

STEP 1: Find the LCD

$$5x - 25 = 5(x - 5)$$

$$x - 5 = x - 5$$

$$\text{LCD} = 5(x - 5)$$

STEP 2: rewrite the numerator

To get from $5(x - 5)$ to $5(x - 5)$ we introduce a factor of 1.

To get from $x - 5$ to $5(x - 5)$, we introduce a factor of 5.

$$\frac{(6x + 5) \times 1}{5(x - 5) \times 1} - \frac{(x + 2) \times 5}{(x - 5) \times 5} = \frac{1 \times (6x + 5)}{5(x - 5)} - \frac{5(x + 2)}{5(x - 5)}$$

$$\frac{1(6x+5)-5(x+2)}{5(x-5)} = \frac{6x+5-5x-10}{5(x-5)} = \frac{x-5}{5(x-5)}$$

STEP 3: Add or subtract numerator

$$\frac{x-5}{5(x-5)} = \frac{1}{5}$$

STEP 4: Simplify completely

g. $\frac{9}{9a^2+6a-8} - \frac{6}{9a^2-4}$

STEP 1: Find the LCD

$$9a^2 + 6a - 8 = (3a - 2)(3a + 4)$$

$$9a^2 - 4 = (3a - 2)(3a + 2)$$

$$\text{LCD} = (3a - 2)(3a + 2)(3a + 4)$$

STEP 2: Rewrite the Numerator

$$\frac{9}{(3a-2)(3a+4)} - \frac{6}{(3a-2)(3a+2)} = \frac{\quad}{(3a-2)(3a+2)(3a+4)} - \frac{\quad}{(3a-2)(3a+2)(3a+4)}$$

To go from $(3a - 2)(3a + 4)$ to $(3a - 2)(3a + 2)(3a + 4)$ we have to introduce a factor of $(3a + 2)$

To go from $(3a - 2)(3a + 2)$ to $(3a - 2)(3a + 2)(3a + 4)$ we have to introduce a factor of $(3a + 4)$

$$\frac{9 \times (3a+2)}{(3a-2)(3a+4) \times (3a+2)} - \frac{6 \times (3a+4)}{(3a-2)(3a+2) \times (3a+4)} = \frac{27a+18}{(3a-2)(3a+2)(3a+4)} - \frac{18a+24}{(3a-2)(3a+2)(3a+4)}$$

$$\frac{27a+18}{(3a-2)(3a+2)(3a+4)} - \frac{18a+24}{(3a-2)(3a+2)(3a+4)} = \frac{27a+18-18a-24}{(3a-2)(3a+2)(3a+4)}$$

STEP 3: Add or subtract numerator

$$\frac{27a + 18 - 18a - 24}{(3a - 2)(3a + 2)(3a + 4)} = \frac{9a - 6}{(3a - 2)(3a + 2)(3a + 4)}$$

$$\frac{9a-6}{(3a-2)(3a+2)(3a+4)} = \frac{3(3a-2)}{(3a-2)(3a+2)(3a+4)} = \frac{3}{(3a+2)(3a+4)}$$

STEP 4: Simplify completely

h. $\frac{4x}{x^2+6x+5} + \frac{x-1}{x^2+5x+4}$

STEP 1: Find the LCD

$$x^2 + 6x + 5 = (x + 5)(x + 1)$$

$$x^2 + 5x + 4 = (x + 4)(x + 1)$$

$$\text{LCD} = (x + 5)(x + 4)(x + 1)$$

STEP 2: Rewrite the Numerator

$$\frac{4x}{(x + 5)(x + 1)} + \frac{x - 1}{(x + 4)(x + 1)} = \frac{4x}{(x + 5)(x + 4)(x + 1)} + \frac{x - 1}{(x + 5)(x + 4)(x + 1)}$$

To go from $(x+5)(x+1)$ to $(x+5)(x+4)(x+1)$ we introduce a factor of $(x+4)$

To go from $(x+4)(x+1)$ to $(x+5)(x+4)(x+1)$ we introduce a factor of $(x+5)$

$$\frac{4x \times (x+4)}{(x+5)(x+1) \times (x+4)} + \frac{(x-1) \times (x+5)}{(x+4)(x+1) \times (x+5)} = \frac{4x^2+16x}{(x+5)(x+4)(x+1)} + \frac{x^2+5x-1x-5}{(x+5)(x+4)(x+1)}$$

STEP 3: Add or subtract the numerator

$$\frac{5x^2 + 20x - 5}{(x + 5)(x + 4)(x + 1)}$$

STEP 4: Simplify completely

$$\frac{5(x^2 + 4x - 1)}{(x + 5)(x + 4)(x + 1)}$$

Complex Fractions:

A complex fraction is defined to be a fraction within a fraction, or a rational expression within a rational expression. Normally, we don't like to leave a fraction or a rational expression in complex form. So this section is devoted to figuring out how to get a complex fraction out of complex form.

There are 2 methods as to which we can simplify complex fractions. We will give one example of the first method, but focus mainly on how to simplify complex fractions using the second method.

Method 1:

To simplify the complex fraction $\frac{\frac{1}{3}}{\frac{4}{9}}$, one way to do this is to rewrite the fraction using division. So that, $\frac{\frac{1}{3}}{\frac{4}{9}} = \frac{1}{3} \div \frac{4}{9} = \frac{1}{3} \times \frac{9}{4} = \frac{9}{12} = \frac{3}{4}$. This is a good method to use should we

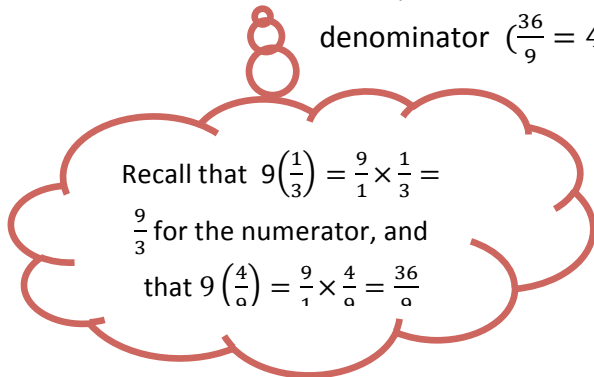
have to simplify complex rational numbers. However, it's a bit more complicated to use this method if we have to simplify complex rational expressions. We use the second method below, which works for either rational numbers or rational expressions:

Method 2:

Using the same example as above, we can simplify $\frac{\frac{1}{3}}{\frac{4}{9}}$ using another method. The second method involves finding the LCD of the whole fraction, and multiplying both the numerator and denominator by that number. Interestingly, once we do this, we will see that the complex bit of the fraction will be forced to cancel, leaving the fraction as a single simplified non-complex fraction!

$\frac{\frac{1}{3}}{\frac{4}{9}}$ First we have to find the LCD of 3, and 9. The LCD of 3 and 9 is 9. We multiply both the top fraction and the bottom fraction by 9. Why is it ok to do this? Well multiplying the fraction by $\frac{9}{9}$ is the same thing as multiplying the fraction by 1. (because $\frac{9}{9}$ simplifies to 1).

So, multiplying $\frac{9}{9} \left(\frac{\frac{1}{3}}{\frac{4}{9}} \right) = \frac{\frac{9}{3}}{\frac{36}{9}}$ now, we simplify the numerator ($\frac{9}{3} = 3$) and denominator ($\frac{36}{9} = 4$) completely, and we get: $\frac{3}{4}$.



To simplify a complex fraction, we use a 3 step process:

STEP 1: Find the Least Common Denominator of the WHOLE Complex Fraction

STEP 2: Multiply the Least Common Denominator by each (term in the) numerator

STEP 3: Simplify Completely

Let's look at some more examples.

Example: Simplify each rational expression completely:

i. $\frac{\frac{2}{5} - \frac{1}{3}}{\frac{5}{6} - 2}$

STEP 1: The LCD of 5, 3, and 6 is 30

$$\frac{30}{30} \left(\frac{\frac{2}{5} - \frac{1}{3}}{\frac{5}{6} - 2} \right) =$$

STEP 2: Multiply the LCD by each numerator

$$\frac{\frac{60}{5} - \frac{30}{3}}{\frac{150}{6} - 60}$$

$$\frac{12 - 10}{25 - 60} = \frac{2}{-35} = -\frac{2}{35}$$

STEP 3: Simplify completely

$$j. \frac{1 + \frac{2}{x}}{\frac{1}{x} - \frac{1}{y}}$$

STEP 1: The LCD of x, x, and y is xy

$$\frac{xy}{xy} \left(\frac{1 + \frac{2}{x}}{\frac{1}{x} - \frac{1}{y}} \right) =$$

STEP 2: Multiply the LCD by each

$$\frac{xy + \frac{2xy}{x}}{\frac{xy}{x} - \frac{xy}{y}}$$

numerator.

$$\frac{xy + 2y}{y - x}$$

STEP 3: Simplify completely

$$k. \frac{\frac{3x+1}{x^2-49}}{\frac{9x^2-1}{x-7}}$$

STEP 1: We notice first that these

fractions are not in terms of factors yet, so we cannot find the LCD. We first have to factor completely in order to find the Least Common Denominator

$$\frac{\frac{3x+1}{x^2-49}}{\frac{9x^2-1}{x-7}} = \frac{\frac{3x+1}{(x-7)(x+7)}}{\frac{(3x-1)(3x+1)}{x-7}}$$

The LCD for $(x-7)(x+7)$ and $(x-7)$ is

$$(x-7)(x+7)$$

$$\frac{(x-7)(x+7)}{(x-7)(x+7)} \left(\frac{\frac{3x+1}{(x-7)(x+7)}}{\frac{(3x-1)(3x+1)}{x-7}} \right) =$$

STEP 2: Multiply the LCD by each

$$\frac{\frac{(x-7)(x+7)(3x+1)}{(x-7)(x+7)}}{\frac{(x-7)(x+7)(3x-1)(3x+1)}{(x-7)}}$$

numerator.

$$\frac{3x+1}{(x+7)(3x-1)(3x+1)} = \frac{1}{(x+7)(3x-1)}$$

STEP 3: Simplify Completely

$$l. \frac{1 - \frac{16}{a^2}}{2 + \frac{9}{a} + \frac{4}{a^2}}$$

STEP 1: The LCD of a^2 , a and a^2 is a^2

$$\frac{a^2 \left(\frac{1 - \frac{16}{a^2}}{2 + \frac{9}{a} + \frac{4}{a^2}} \right)}{a^2 \left(\frac{1 - \frac{16}{a^2}}{2 + \frac{9}{a} + \frac{4}{a^2}} \right)} =$$

STEP 2: Multiply the LCD by each

$$\frac{a^2 - \frac{16a^2}{a^2}}{2a^2 + \frac{9a^2}{a} + \frac{4a^2}{a^2}} = \frac{a^2 - 16}{2a^2 + 9a + 4}$$

numerator

$$\frac{a^2 - 16}{2a^2 + 9a + 4} = \frac{(a-4)(a+4)}{(2a+1)(a+4)} = \frac{a-4}{2a+1}$$

STEP 3: Simplify Completely

$$m. \frac{\frac{1}{b-1} + 1}{\frac{1}{b+1} - 1}$$

STEP 1: The LCD of $(b-1)$ and $(b+1)$ is

$(b-1)(b+1)$

$$\frac{(b-1)(b+1) \left(\frac{1}{b-1} + 1 \right)}{(b-1)(b+1) \left(\frac{1}{b+1} - 1 \right)} =$$

STEP 2: Multiply the LCD by each

numerator

$$\frac{\frac{(b-1)(b+1)}{b-1} + 1(b-1)(b+1)}{\frac{(b-1)(b+1)}{b+1} - 1(b-1)(b+1)} = \frac{(b+1) + 1(b-1)(b+1)}{(b-1) - 1(b-1)(b+1)} = \frac{(b+1) + (b^2-1)}{(b-1) - (b^2-1)} = \frac{b^2+b}{-b^2+b}$$

$$\frac{b^2+b}{-b^2+b} = \frac{b(b+1)}{b(-b+1)} = \frac{b+1}{-b+1}$$

STEP 3: Simplify Completely

$$\text{n. } \frac{\frac{y-1}{y+1} - \frac{y+1}{y-1}}{\frac{y-1}{y+1} + \frac{y+1}{y-1}}$$

STEP 1: The LCD of (y+1) and (y-1) is

$$(y+1)(y-1)$$

$$\frac{(y+1)(y-1) \left(\frac{y-1}{y+1} - \frac{y+1}{y-1} \right)}{(y+1)(y-1) \left(\frac{y-1}{y+1} + \frac{y+1}{y-1} \right)} =$$

STEP 2: Multiply the Numerator by
the LCD

$$\frac{\frac{(y+1)(y-1)(y-1)}{(y+1)} - \frac{(y+1)(y-1)(y+1)}{(y-1)}}{\frac{(y+1)(y-1)(y-1)}{(y+1)} + \frac{(y+1)(y-1)(y+1)}{(y-1)}} = \frac{(y-1)(y-1) - (y+1)(y+1)}{(y-1)(y-1) + (y+1)(y+1)} =$$

$$\frac{(y^2 - 2y + 1) - (y^2 + 2y + 1)}{(y^2 - 2y - 1) + (y^2 + 2y + 1)} = \frac{-4y}{2y^2}$$

$$\frac{-4y}{2y^2} = \frac{-2}{y}$$

STEP 3: Simplify Completely